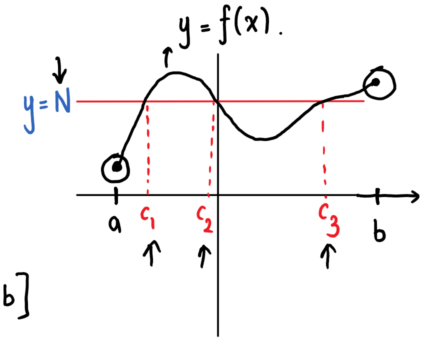
Lecture 6

Tuesday, September 6, 2016 8:52 AM

Intermediate Value Theorem



Suppose f is continuous on the closed interval $[a,b]$ and N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$.

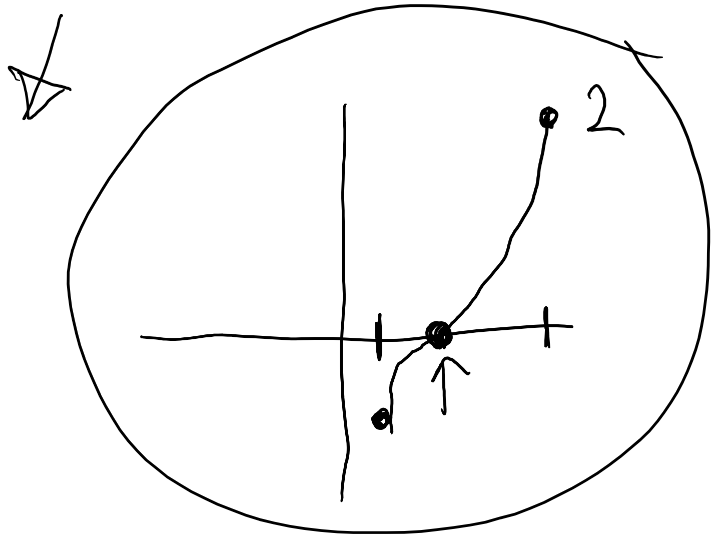
Then, there exists a number c in (a,b) [i.e $a<c<b$] such that $f(c)=N$.

Ex Show that there is a root of the equation $4x^3-6x^2+3x-2=0$ between 1 and 2.

Soln $f(x) = 4x^3-6x^2+3x-2$

WTS There is a c between 1 & 2 ✓
such that $f(c)=0$ ✓

Use I.V.T $a=1, b=2, N=0$



- f is a polynomial and hence is continuous on $[1,2]$
- $f(1) = 4(1)^3-6(1)^2+3\cdot 1-2 =$
  $= 4-6+3-2 = -1 < 0$
- $f(2) = 4(2)^3-6(2)^2+3\cdot 2-2$
  $= 32-24+6-2 = 12 > 0$

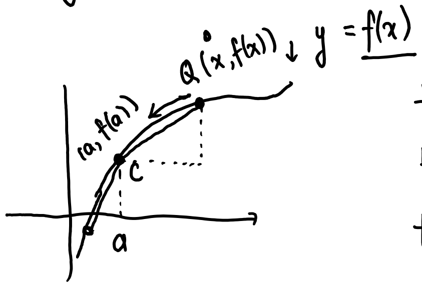
We have that $f(1) < 0 < f(2)$

In particular by IVT there is a number c betn 1 & 2 such that $f(c)=0$

End of 2.5

2.7 Derivatives and Rates of Change

Tangent Line Problem



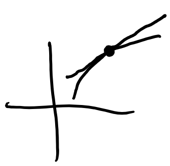
Want to find the slope of the tangent line to the curve $y = f(x)$ at pt $C = (a, f(a))$

Then for pt Q close to C $(x \neq a)$ the slope of the line joining Q and C is given by

$$m_{CQ} = \frac{f(x) - f(a)}{x - a}$$

DEFN The tangent line to the curve $y = f(x)$ at pt $C(a, f(a))$ is the line passing through C w/ slope $m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ ✓

provided the limit exists.



Ex Find the equation of the tangent line to the hyperbola $y = \frac{3}{x}$ at the point $(3, 1)$.

Rmk $m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Set $h = x - a \Rightarrow x = a + h$

$$\boxed{m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Soln $f(x) = \frac{3}{x}$ at the slope of T.L to $y = f(x)$

at $(3, 1)$ is given by
$\uparrow$ $\uparrow$
a $f(a)$

$$m = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - 1 \cdot \frac{3+h}{3+h}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - \frac{3+h}{3+h}}{h}$$

$$= \lim_{h \to 0} \frac{3 - (3+h)}{3+h} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-h}{3+h} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-1}{3+h} = -\frac{1}{3}$$

Point Slope formula : $y - f(a) = m(x - a)$

$$\boxed{y - 1 = -\frac{1}{3}(x - 3)}$$

$$\boxed{y - 1 = -\frac{1}{3}(x-3)}$$

<u>DEF</u> The derivative of func f at pt a, denoted by $f'(a)$ is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

provided limit exists.

The derivative of a function $f(x)$ is also a func, denote by $f'(x)$, where

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

<u>Ex</u> Find the derivative of the function $f(x) = \underline{x^2 - 8x + 9}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\overbrace{(x+h)^2 - 8(x+h) + 9}^{f(x+h)} - \overbrace{(x^2 - 8x + 9)}^{f(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{8x} - 8h + \cancel{9} - \cancel{x^2} + \cancel{8x} - \cancel{9}}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 8h}{h}$$

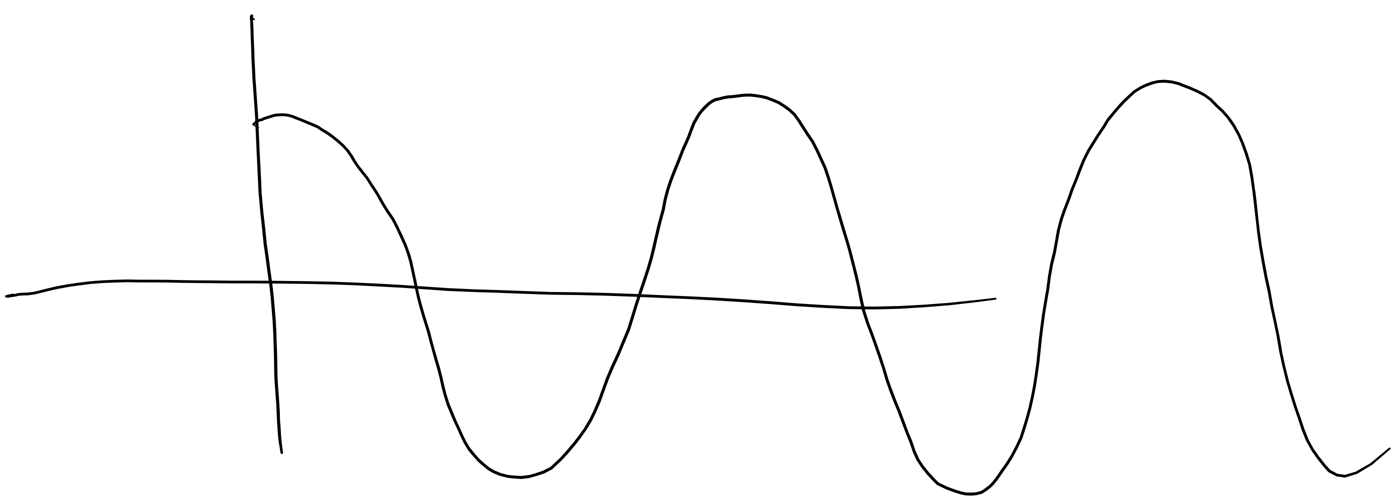
$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h-8)}{\cancel{h}}$$

$$f'(x) = \underline{\underline{2x - 8}}$$

Ex Find the slope of the T.L to the curve $y = \underline{x^2 - 8x} + 9$ at $x = 3$.

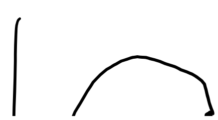
DEF The slope of the T.L. to the curve $y = f(x)$ at pt $(a, f(a))$ is $m = f'(a)$.

Soln $m = f'(3) = 2.3 - 8 = -2$



$$\lim_{x \to a} f(x) = f(a)$$

Cannot



$x \to a$